

POWER AND ENERGY DISSIPATION IN SUBSEQUENT RETURN STROKES AS PREDICTED BY A NEW RETURN STROKE MODEL

Vernon Cooray
Institute of High Voltage Research
(A department at Uppsala University)
Uppsala, Sweden

ABSTRACT

Recently, Cooray [1] introduced a new return stroke model which is capable of predicting the temporal behaviour of the return stroke current and the return stroke velocity as a function of height along the return stroke channel. We have employed this model to calculate the power and energy dissipation in subsequent return strokes. The results of these calculations are presented in this paper. The main conclusions of this paper are the following. (a) A large fraction of the total energy available for the dart leader-subsequent stroke process is dissipated in the dart leader stage. (b) The peak power per unit length dissipated in a subsequent stroke channel element decreases with increasing height of that channel element from ground level. For a given channel element the peak power dissipation increases with increasing current in that channel element. (c) The peak electrical power dissipation in a typical subsequent return stroke is about 1.5×10^{11} W. (d) The energy dissipation in a subsequent stroke increases with increasing current in the return stroke channel and for a typical subsequent stroke the energy dissipation per unit length is about 5.0×10^3 J/m.

INTRODUCTION

Recently Cooray [1] introduced a new return stroke model which is capable of predicting the temporal behaviour of the return stroke current and the return stroke velocity as a function of height along the return stroke channel. The basic concepts used in this return stroke model are the following. The dart leader current travels along the central core of the channel depositing charge along the channel. Due to the high lateral field, this charge expands outward giving rise to a corona sheath. At the point of contact with ground all the charge is concentrated on the central core and, due to the development of the corona sheath, the charge on the core decreases rapidly upwards. As the return stroke propagates up, the charge on the central core and corona sheath discharge into the channel. This gives rise to current components which subsequently travel along the highly conducting return stroke channel to ground. The total current at any height is due to the sum of sheath current and core current from levels above that height. The amplitude of the core current decreases rapidly upwards and the duration of the sheath current is assumed to be equal to the time for the streamers to propagate into the space charge region. The velocity of the return stroke along the channel is calculated as a function of current parameters and channel temperature. Most of the predictions of this model concerning the radiated electromagnetic fields and the variation of return stroke velocity, current risetime, peak current amplitude, and peak current derivative along the return stroke channel are substantiated by recent experimental observations.

In the study reported in this paper we have employed this return stroke model to calculate the power and energy dissipation during subsequent return strokes. First, from the charge and

current distribution along the return stroke channel the electric field inside the channel as a function of time is calculated. This was combined with the current waveform along the channel to calculate the power and energy dissipation during subsequent return strokes. In this paper we will present results to show how the power dissipation along the channel is changing as a function of height from ground level and the variation of power and energy dissipation as a function of peak current at ground level.

POWER DISSIPATION ALONG THE CHANNEL

The current in the return stroke channel at a point situated 10 m from ground level is shown in figure 1(a). The current waveform rises to its peak value, which is about 10 kA, in about 0.3 μ s. The axial electric field, caused by the current and charge distribution along the return stroke channel, in the same channel element as a function of time is shown in figure 1(b). The method of calculation of the axial field in the channel element is similar to that described by Cooray et al. [2]. In calculating this axial field the radius of the return stroke channel was assumed to be 0.001 m. The value of 0.001 m may be a good approximation for the radius of the return stroke channel at the initial stages of the discharge but at later times the channel radius may be of the order of 0.01 m. Note that the electric field in the channel has a peak value of about 3.5×10^5 V/m and it decreases to 4×10^3 V/m in about 3.0 μ s. This low value of the electric field is probably associated with the arc phase of the discharge. For example, an arc channel in air with a current of 10 kA has a electric field in the range of about 10^3 V/m [3]. Our calculations show that the resistance of the channel decreases with time reaching a minimum value of 0.5 ohm/m in about 3.0 microsecond.

The power dissipation in the channel element as a function of time is shown in figure 1(c). Note that the peak power dissipation in this channel element is about 2.0×10^9 W/m. After the initial peak the power decreases to about 5×10^7 W/m. Our calculations show that the return stroke can maintain this low level of power dissipation for several tens of microseconds. This low level of power dissipation is probably associated with the arc phase of the discharge. A similar procedure can be applied to calculate the power dissipation in channel elements situated at different heights along the channel. For example figure 2 shows the power dissipation, as a function of time, in channel elements situated at 10 m, 100 m and 500 m from ground level. Note that the width of the of the initial peak in the power curve increases and the peak power decreases with increasing height from ground level. The peak power generated by channel elements as a function of their height from ground level is shown in figure 3 for three peak values of current, i.e. 10 kA, 20 kA, and 30 kA , at ground level. Note that over the first 100 m from ground the peak power dissipation in channel elements decreases by almost a factor of three. This decrease in peak power is caused by the decrease in current amplitude and the increase in current risetime with increasing height from ground level.

As can be seen from the data in figure 3, the peak power dissipation in a channel element at a given height increases with increasing peak value of the current amplitude at ground level. The relationship between these two parameters for a channel element at ground level is shown in figure 4. This relationship can be represented approximately by the formula

$$P_p = 4.0 \times 10^7 I_p^{1.7} \quad (1)$$

where I_p is the peak current in kA at ground level and P_p is the peak power in W/m generated by the channel element. The relationship given in (1) is drawn in figure 4 by a dashed line. Of course it is possible that the relationship between these two quantities is not the same at other

heights along the channel since the shape of the current and the velocity of the return stroke is changing along the channel.

The data given above correspond to the power dissipation in a single channel element situated at a given point along the channel. However, the return stroke is composed of large number of channel elements and the number of channel elements contributing to the power dissipation increases with increasing time due to the geometrical growth of the return stroke channel. Therefore, the total power dissipation in the return stroke is a function of the geometrical growth of the return stroke channel and the time development of the power in each individual segment of the channel. The total power dissipation in the return stroke channel as a function of time for a 10 kA current at ground level is shown in figure 5. The peak power dissipation in this return stroke is about 9×10^{10} W. Note that the power reaches its peak value in about $7.5 \mu\text{s}$ and our calculations show that the height of the return stroke channel when the power reaches its peak value is about 1 km. Figure 6 shows the variation of peak power dissipation in return strokes as a function of peak current. Again observe that the peak power dissipation in subsequent return strokes increases with increasing peak current.

ENERGY DISSIPATION IN RETURN STROKES

The temporal variation of the power dissipation can be integrated to calculate the energy dissipation along the return stroke channel as a function of time. Figure 7 shows the energy dissipation, as a function of time, in a channel element situated at 10 m from ground level. Note that the energy dissipation along in the channel element increases rapidly initially which is followed by a slow increase. The peak energy dissipation in the channel element (not shown in the diagram) is about 4.0×10^3 J/m. We have also calculated the energy dissipation in channel elements close to ground as a function of peak return stroke current. The results are shown in figure 8. Note that the energy dissipation in the return stroke channel increases with peak current. For a 30 kA peak current the energy dissipation in the return stroke channel is about 2.8×10^4 J/m and for a 50 kA current it is about 8.0×10^4 J/m. The relationship between the total energy dissipation, per unit length, in channel elements close to ground and the peak current at ground level can be represented approximately by the formula

$$E = 43.2 I_p^{1.9} \quad (2)$$

where E is the total energy, in J/m, dissipated in the channel and I_p is the peak current, in kA, in the channel. This formula is depicted in figure 8 by a dashed line. Note that the energy dissipation in the return stroke channel increases approximately as the square of the peak current.

DISTRIBUTION OF POWER AND ENERGY DISSIPATION IN SUBSEQUENT RETURN STROKES

As we have shown above the energy and the power that will dissipate in a return stroke depend on the peak current in the return stroke channel. On the other hand as shown by Cooray [4] the return stroke model predicts an approximate linear relationship between the peak return stroke current and the peak return stroke radiation field. Therefore, if the distribution of peak radiation fields generated by subsequent return strokes are known, the relationships given in the earlier section can be used to obtain the distribution of power and energy dissipation in subsequent return strokes. The best data set available at present on the distribution of peak radiation fields from subsequent strokes was obtained by Master et al. [5]. We have used this distribution to calculate the distribution of energy and power dissipation in return strokes. The results are

shown in figure 9(a) and 9(b). Note that the 50% values of the energy and power dissipation in subsequent return strokes are 5.0×10^3 J/m and 1.5×10^{11} W/m respectively.

DISCUSSION

Several theoretical simulations of the energy dissipation in lightning return strokes have been made by several researchers [6,7,8,9,10]. In an investigation conducted by Hill [6] power and energy dissipation due to a 21 kA current was estimated. The risetime of the current waveform was 8.5 μ s. The peak power dissipation in the channel was calculated to be 1.5×10^9 W/m and the total energy dissipation in the channel was estimated to be about 1.5×10^4 J/m. Plooster [7] estimated that a 20 kA current with a 5 μ s risetime will dissipate about 2.5×10^3 J/m of energy in the return stroke channel. More recently Paxton et al. [10] estimated that a 20 kA return stroke current with a 5 μ s risetime will dissipate about 4.0×10^3 J/m of energy in the return stroke channel. Our analysis shows that the peak power dissipation in return strokes depends on the risetime of the current waveform. The results in figure 3 show that the peak power dissipation in a return stroke channel element close to ground with a 20 kA peak current is about 7×10^9 W/m. Furthermore, the risetime of the current waveform close to ground level is about 0.3 μ s. However, the risetime of the current waveform increases with height and at heights of about 500m from ground level the risetime of the current waveform is about 5 μ s. Calculations show that the peak power dissipation caused by a 21 kA current at this level is about 1.0×10^9 W/m. This value agrees with the peak power estimated by Hill [6]. Furthermore, our estimates of energy dissipation are in general agreement with the values reported in the literature.

To the best of our knowledge no experimental observations of electrical energy dissipation in subsequent return strokes are available in the literature. In the case of first return strokes the only available information is due to Krider et al. [11]. In that investigation the energy dissipated in a single stroke flash was estimated to be 2.3×10^5 J/m. The current in the return stroke was unknown. On the other hand several researchers have attempted to calculate the energy dissipation in return strokes by electrostatic considerations [11,12,13,14]. According to these electrostatic considerations the potential of the cloud is of the order of $10^8 - 10^9$ V. Now, according to the return stroke model of Cooray [1] the total charge brought down by a 10 kA subsequent return stroke is about 0.5 C. If we assume that the length of the return stroke channel is about 7 km and the potential of the cloud is about 5×10^8 V then the energy per unit length available for the return stroke process would be of the order of 3.5×10^4 J/m. However, the calculated value of 4.0×10^3 J/m is about an order of magnitude less than this expected value. If the calculated value is correct then some of the energy available for the return stroke-dart leader process must dissipate in a process other than the return stroke. It is possible that some of this energy will dissipate in the dart leader stage but, unfortunately, no information is available in the literature concerning the energy dissipation in the dart leader stage. In order to investigate this apparent discrepancy we have used the following simplified model to estimate the energy dissipation in the two stages i.e. dart leader and return stroke. The two charge centers, i.e. negative and positive, in the cloud were assumed to be spherical in shape and each charge center was assumed to carry 40 C of charge. The radius of the charge center was calculated by assuming that the outer boundary of the charge center is at breakdown electric field. Now, due to the decrease in pressure at cloud height the breakdown field may be a factor of 2 lower than that at atmospheric pressure and due to the presence of water drops the breakdown field may decrease further by a factor of about 3 [15]. Therefore, the radius of the charge centers will be of the order of 1 km. The height of the center of negative charge was assumed to be at 8 km and the

height of the positive charge center was assumed to be at 12 km. The situation is shown in figure 10(a). Now, according to the return stroke model, just before the return stroke all the charge that will be neutralised in the return stroke is distributed over the dart leader channel. The charge density (denoted by ρ) along the channel was assumed to decrease exponentially with height with a decay height constant of 5000 km. The diameter of the dart leader channel was calculated by assuming that the outer boundary of the charge cylinder is at breakdown electric field. The situation is shown in figure 10(b). The total charge on the leader channel is 0.5 C which is the value corresponding to a 10 kA peak current. During the return stroke the charge stored on the leader channel is brought to ground and at the end of the return stroke the situation is similar to that depicted in figure 10(c). Using this configuration we have calculated the energy dissipation during the dart leader stage i.e. the energy difference in the situations 10(a) and 10(b) and the energy dissipation during the return stroke stage i.e. the energy difference in the situations 10(b) and 10(c). In these calculations the effect of the earth was taken into account by the method of images. The results show that the energy dissipation during the dart leader stage is about 2.0×10^4 J/m and the energy dissipation during the return stroke is about 2.6×10^3 J/m. This shows that about 90% of the energy available for the dart leader- return stroke process will dissipate in the dart leader stage. The value calculated for the energy dissipation in the return stroke i.e. 2.6×10^3 J/m is similar to that calculated by using the return stroke model. Now, the total energy dissipation in bringing down 0.5 C of charge to ground is about 2.3×10^4 J/m and since the length of the leader channel is 7.5 km the total energy dissipation during the dart leader-return stroke process is 1.7×10^8 J. This energy value corresponds to a discharge to ground of 0.5 C from an initial potential difference of 3.5×10^8 V. This value agrees with the estimations of cloud potential obtained by other researchers.

CONCLUSIONS

In this paper we have calculated the energy and power dissipation in subsequent return strokes by using a recently introduced return stroke model. For a given return stroke current the highest power is dissipated in channel elements close to ground and the peak power dissipation in a given channel element decreases with increasing height of that channel element from ground level. For example, our results show that, for a 10 kA peak current, the peak power dissipation in a channel element at ground level is about 2.0×10^9 W/m and the peak power dissipation in a channel element at 100 m from ground level is about 6×10^8 W/m. The total power dissipation in the overall return stroke channel reaches its peak when the length of the return stroke channel is about a kilometer and for a typical subsequent stroke peak total power dissipation is about 1.5×10^{11} W. The energy dissipation in a typical subsequent return stroke is about 5.0×10^3 J/m. Both the energy and the power dissipation in the return strokes increase with increasing peak current. The calculations presented in this paper show that most of the energy available for the dart leader-return stroke process is dissipated in the dart leader stage.

ACKNOWLEDGEMENTS

The research work reported here was supported by a grant (No. E-EG 1448-303) to the author from the Swedish Natural Science Research Council.

REFERENCES

- [1] Cooray, V., A return stroke model, Proceedings of the 1989 International Conference on Lightning and Static Electricity, pp. 1A.31-1A.39, University of Bath, United Kingdom, September, 1989.

- [2] Cooray, V., V. P. Idone and R. E. Orville, Velocity of self propagating discharge as a function of current parameters with special attention to return strokes and dart leaders, UURIE: 207-88, Institute of High Voltage Research, University of Uppsala, Sweden.
- [3] King, L. A., The voltage gradient of the free burning arc in air or nitrogen, Proceedings of the International Conference on Ionization Phenomena in Gases, pp. 871-877, Munich, 1961.
- [4] Cooray, V., Relationship between different return stroke parameters as predicted by a new return stroke model, Proceedings of the International Conference on Lightning Protection, 2.11P/1-2.11P/7, Interlaken, Switzerland, 1990.
- [5] Master, M. J., M. A. Uman, W. H. Beasley and M. Darveniza, Lightning induced voltages on power lines: Experiment, IEEE Trans. PAS, PAS-103, 2519-2529, 1984.
- [6] Hill, R. D., Channel heating in return stroke lightning, J. Geophys. Res., 76, pp. 637-645, 1971.
- [7] Plooster, M. N., Numerical model of the return stroke of the lightning discharge, Phys. Fluids, 14, pp. 2124-2133, 1971.
- [8] Hill, R. D., Energy dissipation in lightning, J. Geophys. Res., 82, pp. 4967-4968, 1977.
- [9] Hill, R. D., A survey of lightning energy estimates, Reviews of Geophys. and Space Phys., 17, pp. 155-164, 1979.
- [10] Paxton, A. H., R. L. Gardner and L. Baker, Lightning return stroke. A numerical calculation of the optical radiation, Phys. Fluids, 29, pp. 2736-2741, 1986.
- [11] Krider, E. P., G. A. Dawson and M. A. Uman, Peak power and energy dissipation in a single-stroke lightning flash, J. Geophys. Res., 73, pp. 3335-3339, 1968.
- [12] Wilson, C. T. R., Investigations on lightning discharges and on the electric field of thunderstorms, Phil. Trans. Roy. Soc. London. Ser. A. 221, pp. 73-115, 1920.
- [13] Malan, D. J., Physics of Lightning, pp. 75, English University Press, London, 1963.
- [14] Connor, T. R., The 1965 ARPA-AEC joint lightning study at Los Alamos, Rep. LA-3754, vol. 1, Los Alamos Sci. Lab., Los Alamos, New Mexico, 1967.
- [15] Uman, M. A., Lightning, McGraw-Hill, New York, 1969.

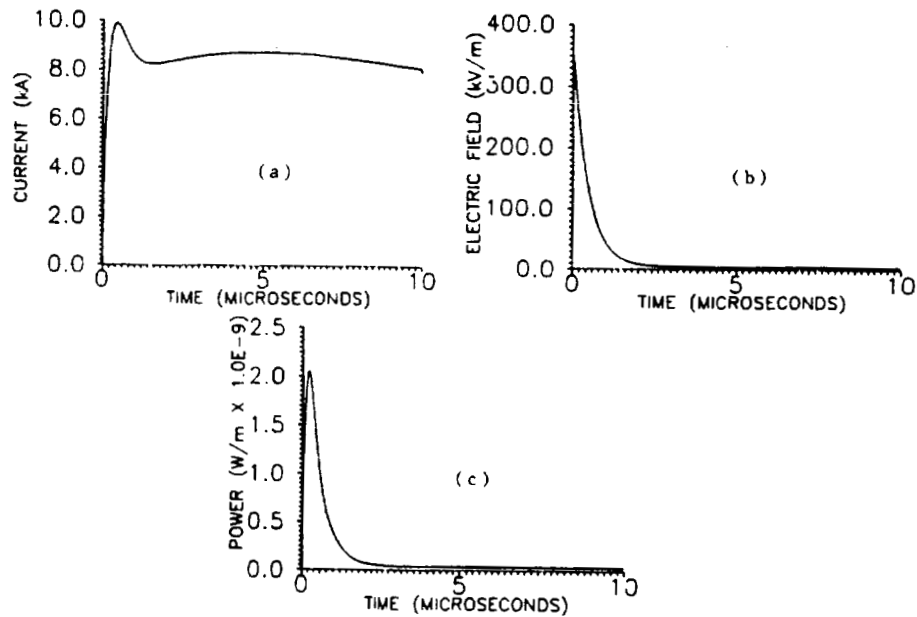


Fig.1 (a) The current, (b) the axial electric field and (c) the power dissipation as a function of time in a subsequent stroke channel element situated at 10 m from ground level. The peak current in the return stroke channel is 10 kA. The time is measured from the beginning of the current waveform in the channel element.

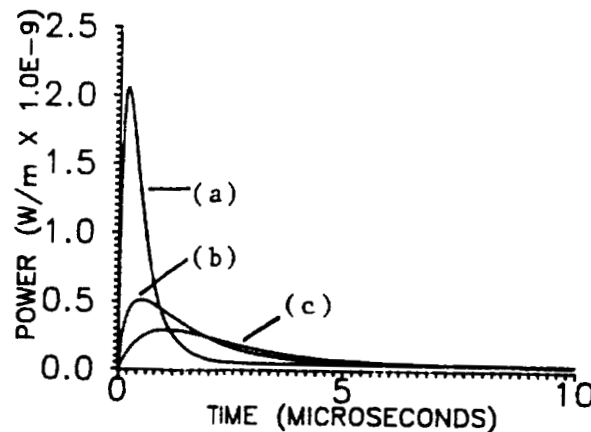


Fig.2 Power dissipation, per unit length, as a function of time in channel elements situated at (a) 10 m, (b) 100 m, and (c) 500 m from ground level. The peak current in the subsequent return stroke is 10 kA. Note that the time delay between the initiation of power dissipation in different channel elements is not shown in the diagram.

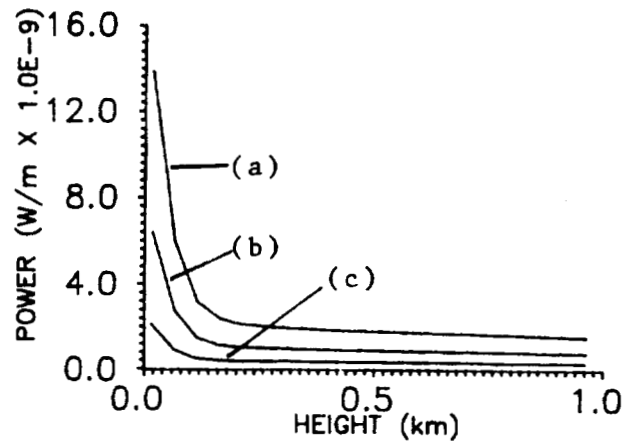


Fig.3 Peak power dissipation, per unit length, in a channel element as a function of it's height from ground level. The peak current in the return stroke channel is (a) 30 kA, (b) 20 kA, and (c) 10 kA.

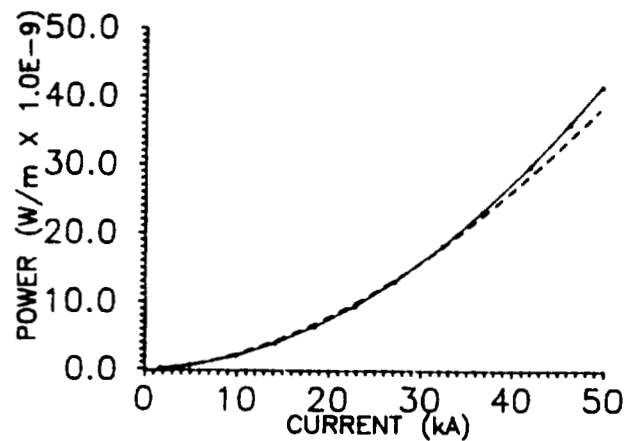


Fig.4 Peak power dissipation, per unit length, in a channel element at ground level as a function of the peak current (solid line). The relationship between these two parameters can be approximated (dashed line) by the equation $P_p = 4.0 \times 10^7 I_p^{1.7}$ where I_p is the peak current in kA at ground level and P_p is the peak power generated by the channel element.

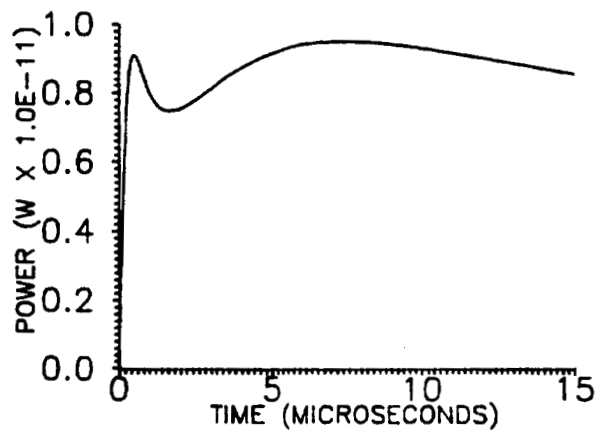


Fig.5 The total power dissipation as a function of time in a return stroke with a 10 kA peak current at ground level.

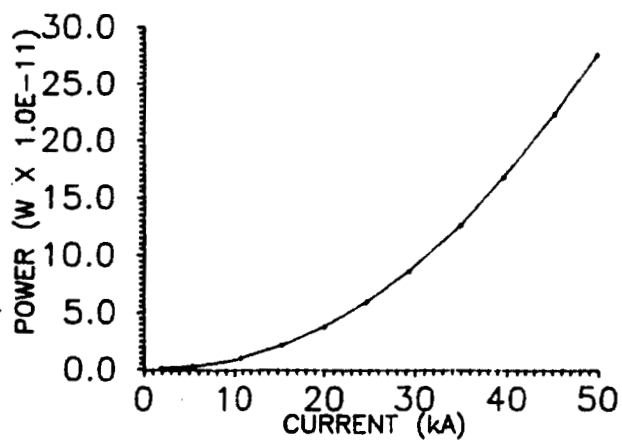


Fig.6 Peak total power dissipation in subsequent return strokes as a function of peak current at ground level.

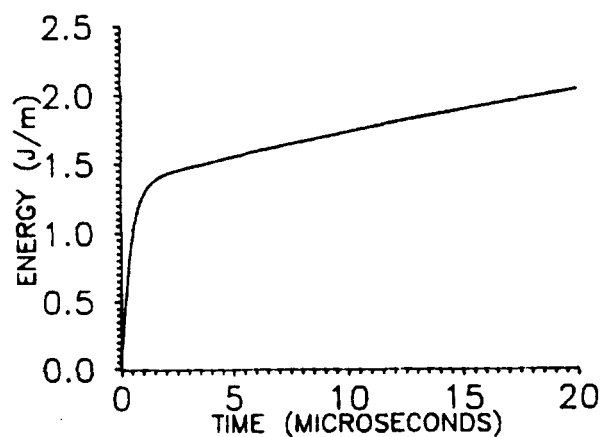


Fig.7 Energy dissipation, per unit length, in a channel element at 10 m from ground level as a function of time. The peak current in the return stroke channel is 10 kA.

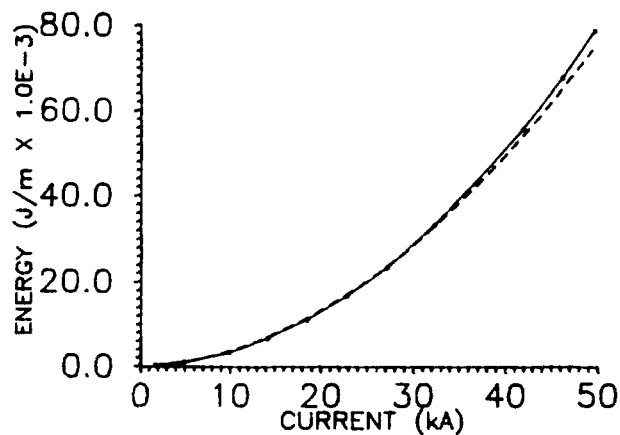


Fig.8 The total energy dissipation, per unit length, in channel elements close to ground as a function of return stroke peak current (solid line). The relationship between these two parameters can be approximated (dashed line) by the equation $E = 43.2 \times I_p^{1.9}$ where I_p is the peak current in kA at ground level and E is the peak power generated by the channel element.

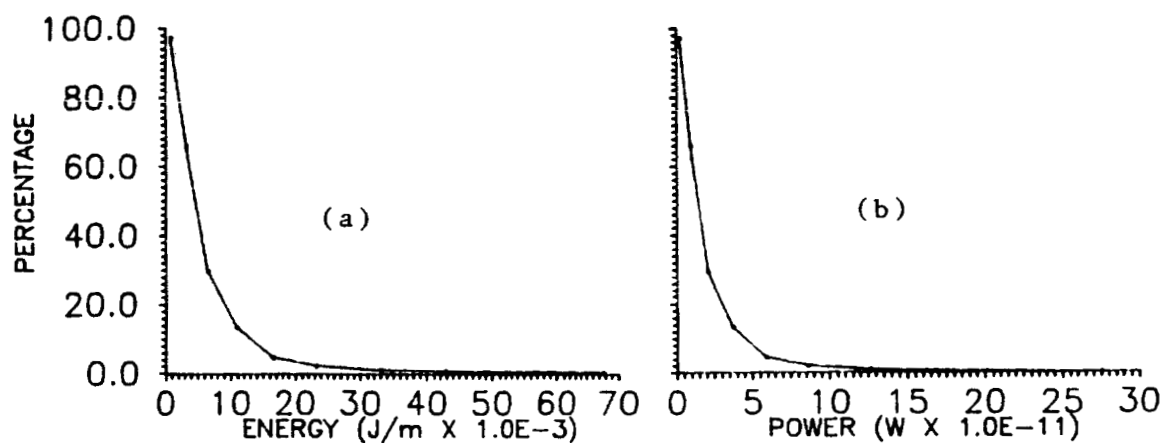


Fig.9 (a) Distribution of the energy dissipation in subsequent return strokes and (b) distribution of the total power dissipation in subsequent return strokes derived from the distribution of peak electric radiation fields.

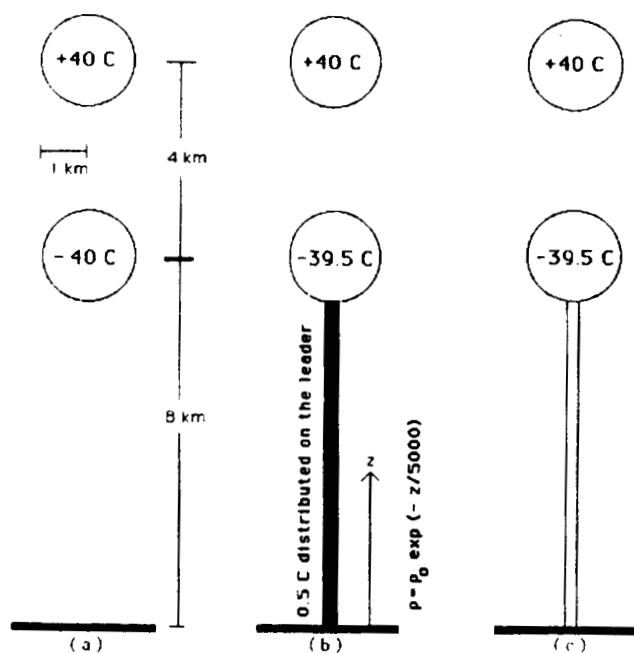


Fig.10 Analysis of the energy dissipation in return strokes by using a simplified model of the thunder cloud. (a) Situation just before the initiation of dart leader. (b) Situation just before the initiation of return stroke. (c) Situation just after the end of return stroke.